Measurement Uncertainty for Weight Determinations in Seized Drug Analysis

NOTE: Changes have been incorporated throughout Revision 2 to include an additional example of Dynamic Weighing and a more thorough discussion of correlation.

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Introduction

The following demonstrates the application of an uncertainty budget approach for weight determinations. The factors described in Part IV C, Section 4, are considered. It is assumed that the value being reported is the conventional mass and final results are rounded to the precision of the balance. The term "weight" is used interchangeably with "conventional mass," the quantity typically reported. Definitions for the statistical terms used can be found in the glossary or references listed below. The references also contain additional examples and detailed information regarding estimation of uncertainty.

Sequential weighing events (such as a tare followed by a net mass determination) may be uncorrelated, correlated, or partially correlated. In practice, the exact value of correlation coefficients is difficult to establish. Therefore, the methods illustrated in this document (SD-3) represent a conservative approach in which the uncertainty is likely to be overestimated. Where applicable, references are provided for labs that elect to establish their own correlation values.

Weighings can be obtained using dynamic or static operations. A dynamic weighing process involves placing a weighing vessel on a balance, taring the balance, and adding material immediately to the weighing vessel without removing it from the balance. A static weighing process involves removal of the tared weigh vessel, filling with material, and then returning to the balance to obtain the net weight. Examples are included in this document for both scenarios.

The following examples should not be directly applied to methodology used without first considering the specific purpose of a method, and its relevant operational environment and the operational capabilities and parameters of the balance.

A Example 1: Dynamic Weighing of a Single Item Using a Budget Table

Scenario: A laboratory must determine the weight of a white powder, which appears to weigh approximately 30 g, received in a plastic bag. This one bag is considered to be one item. The decision is made to weigh the material using a balance with a maximum capacity of 3100 g. The following conditions apply: the operator is competent on the use of the balance; the balance is calibrated and certified as per established laboratory protocols; the balance is being used above the defined minimum balance load; and the balance is performing within the manufacturers’ specifications. The balance operates in a controlled environment using a draft shield with ambient temperature varying ±5 °C.

The weight is determined as follows: A weigh vessel is placed on the balance and tared. The analyst immediately transfers the contents of the plastic bag to the tared weigh vessel without removing it from the balance and records the net weight of the material. The entire operation is considered as a single weighing event (a dynamic weighing).

The net weight obtained for the powder is 30.03 grams.

A.1 Factors contributing to weight measurement uncertainty

The factors considered in estimating the measurement uncertainty include readability; repeatability; linearity; buoyancy; temperature effects; uncertainty from balance calibration report; and sample loss in transfer. Although in rare cases sample losses could be large, the
inability to accurately estimate the uncertainty due to sample losses is not deemed a major concern since sample losses always result in underestimation of the quantity of a substance being weighed. Therefore, uncertainty due to sample loss is not included in any of the uncertainty computations given in this document (SD-3).

Buoyancy is difficult to account for in seized drug cases because the density of the material being weighed must be known. However, for material that has a lower density than steel (8.0 g/cm³), the bias imparted is always negative and the weight displayed by the balance will be less than the true weight of the material. Ignoring buoyancy contributes a small systematic error that represents no more than 0.1% bias to the weight. Therefore, buoyancy corrections are not made in any uncertainty computations shown in this document (SD-3).

Based on the current calibration and performance certification for the balance and given that the balance is operating within specifications, other factors (e.g., environmental, static electricity, corner loading) are deemed insignificant in this example. Laboratories should examine their balances, calibration reports, methods, circumstances, and applications to determine which factors are significant and which are insignificant for their particular application.

The factors deemed significant in this example are expressed in the budget table to follow.
### A.2 Uncertainty Budget Table

<table>
<thead>
<tr>
<th>Factors</th>
<th>Value (x)</th>
<th>Standard uncertainty (u), g</th>
<th>Distribution</th>
<th>Index (Relative contribution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Readability&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.01 g</td>
<td>$\frac{x}{\sqrt{3}} = \frac{0.01}{\sqrt{3}} = 0.0057$</td>
<td>Rectangular</td>
<td>$\frac{0.00577^2}{0.00313} \times 100 = 10.6%$</td>
</tr>
<tr>
<td>Repeatability (s)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.0101 g</td>
<td>0.0101</td>
<td>Normal</td>
<td>32.5%</td>
</tr>
<tr>
<td>Linearity&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.02 g</td>
<td>$\frac{x}{\sqrt{3}} = \frac{0.02}{\sqrt{3}} = 0.0116$</td>
<td>Rectangular</td>
<td>42.9%</td>
</tr>
<tr>
<td>Temperature coefficient&lt;sup&gt;e&lt;/sup&gt;</td>
<td>6 ppm/°C (6x10^-6 g /°C)</td>
<td>$\frac{x}{\sqrt{3}} = \frac{6 \times 10^{-6} g}{\sqrt{3}} \times 10°C \times 30.03g}{\sqrt{3}} = 0.00104$</td>
<td>Rectangular</td>
<td>0.3%</td>
</tr>
<tr>
<td>Uncertainty from balance calibration report (U, coverage factor k=2)&lt;sup&gt;e,f&lt;/sup&gt;</td>
<td>0.0131 g</td>
<td>$\sqrt{\frac{U}{k}} = \sqrt{\frac{0.0131}{2}} = 0.00655$</td>
<td>Normal</td>
<td>13.7%</td>
</tr>
<tr>
<td><strong>Subtotal ($\sum u_i$):</strong></td>
<td></td>
<td>0.0350</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Subtotal ($\sum (u_i)^2$):</strong></td>
<td></td>
<td>0.000313</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> $\frac{u^2}{\sum u_i^2} \times 100$, This value is used to determine which terms are significant.

<sup>b</sup> Obtained from the current calibration and performance certification for the balance and assumes that the balance has a single readability range.

<sup>c</sup> Determined empirically in the laboratory.

<sup>d</sup> This value is the maximum permitted deviation across the mass range of the balance.

<sup>e</sup> Value obtained from manufacturer specifications.

<sup>f</sup> A conservative approach would involve the measurement of uncertainty of the balance calibration at the upper working mass range.
A.3 Calculation of combined standard uncertainty

Considering all factors noted above (A.2) as uncorrelated for a single weighing event, the combined standard uncertainty can be expressed mathematically as:

\[ u_c(\text{single weighing event}) = \sqrt{u(\text{read})^2 + u(\text{repeat})^2 + u(\text{linear})^2 + u(\text{bal cal})^2} \]

where \( u \) is the standard uncertainty and \( u_c \) is combined standard uncertainty. The factor \( u(\text{temperature coefficient}) \) is not included in the combined uncertainty due to its minimal relative contribution to the total standard uncertainty.

The combined standard uncertainty is:

\[ u_c(\text{single weighing event}) = \sqrt{(0.00577 \, g)^2 + (0.0101 \, g)^2 + (0.0116 \, g)^2 + (0.00655 \, g)^2} = 0.0176 \, g \]

A.4 Calculation of expanded uncertainty

The expanded uncertainty is expressed mathematically as:

\[ U = k \times u_c \]

Using a coverage factor \( k = 2 \) (confidence level of approximately 95%, assuming the net mass follows a normal distribution):

\[ U = 2 \times 0.0176 \, g = 0.0352 \, g \]

Using a coverage factor \( k = 3 \) (confidence level of approximately 99% assuming the net mass follows a normal distribution):

\[ U = 3 \times 0.0176 \, g = 0.0528 \, g \]

A.5 Results

A.5.1 Net Weight: 30.03 g ± 0.04 g (\( k = 2 \))

A.5.2 Net Weight: 30.03 g ± 0.05 g (\( k = 3 \))

---

1 The approximate confidence levels given in this document (SD-3) assume that the quantities for which expanded uncertainties are being computed each approximately follow a normal distribution. If this assumption does not hold, the actual confidence level attained for these uncertainty intervals may be lower or higher than the desired levels of 95% or 99%.
B  Example 2: Static Weighing of a Single Item Using a Budget Table

Scenario: The scenario is the same as in Example 1.

The weight is determined as follows: A weigh vessel is placed on the balance and tared. It is then removed from the balance and the powder is transferred to the weigh vessel, which is placed on the balance and a reading obtained (a static weighing).

The net weight obtained for the powder is 30.03 grams.

B.1 Factors contributing to weight measurement uncertainty

The factors are the same as in Example 1.

B.2 Uncertainty Budget Table

The uncertainty budget table is the same as in Example 1. The factor u (temperature coefficient) is not included in the combined uncertainty due to its minimal relative contribution to the total standard uncertainty.

B.3 Calculation of combined standard uncertainty

The combined standard uncertainty for a single weighing event is the same as in Section A, Example 1:

\[
u_{c}(\text{single weighing event}) = \sqrt{(0.00577 \text{ g})^2 + (0.0101 \text{ g})^2 + (0.0116 \text{ g})^2 + (0.00655 \text{ g})^2} = 0.0176 \text{ g}
\]

In this case, the calculation of total uncertainty for the net mass is:

\[
u_{\text{total}} = \text{number_of_items} \ast \sqrt{2 - 2r} \ast u_{c}(\text{single weighing event})
\]

where \( r \) is the correlation coefficient between the uncertainty associated with the tare and the uncertainty associated with the weighing of the material. Because this is a static weighing process, two separate weighing events are considered, the taring of the weigh vessel and the weighing of the material.

In practice, the value of \( r \) is difficult to determine.\(^2\) In this example, the most conservative approach was taken by assigning \( r \) the value of -1. This will likely result in an overestimation of the uncertainty.

\(^2\) If the two weighing events are completely uncorrelated, \( r = 0 \). Under these conditions, the expression simplifies to:

\[
u_{\text{total}} = \text{number_of_items} \ast \sqrt{2} \ast u_{c}(\text{single weighing event})
\]

\[
u_{\text{total}} = 1 \text{ item} \ast \sqrt{2} \ast 0.0176 \text{ g} = 0.0249 \text{ g}
\]
\[ u_{\text{total}} = \text{number_of_items} \times 2 \times u_c(\text{single weighing event}) \]
\[ u_{\text{total}} = 1 \text{ item} \times 2 \times 0.0176 \text{ g} = 0.0352 \text{ g} \]

Alternatively, the laboratory may elect to consider the two weighing events as completely uncorrelated or select another value for \( r \).

**B.4 Calculation of expanded uncertainty**

The expanded uncertainty is expressed mathematically as:

\[ U = k \times u_{\text{total}} \]

Using a coverage factor \( k = 2 \) (confidence level of approximately 95% assuming the net mass follows a normal distribution):

\[ U = 2 \times 0.0352 \text{ g} = 0.0704 \text{ g} \]

Using a coverage factor \( k = 3 \) (confidence level of approximately 99% assuming the net mass follows a normal distribution):

\[ U = 3 \times 0.0352 \text{ g} = 0.1056 \text{ g} \]

**B.5 Results**

**B.5.1** Net Weight: 30.03 g ± 0.07 g \((k = 2)\)

**B.5.2** Net Weight: 30.03 g ± 0.11 g \((k = 3)\)

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3 If not known from relevant references in the metrology literature or experience, the value of the correlation coefficient \( r \) may be determined empirically by performing an experiment with mass standards that approximate the gross and tare masses of the seized drugs on the balance. If the correlation between the gross and tare masses is based on common corrections applied in the computation of each mass, the value of the correlation coefficient \( r \) can be obtained using the methods in Section F.1.2.3 of the GUM [E.3.2].
**C. Example 3: Static Weighing of a Single Item Using Control Chart Data in a Budget Table**

**Scenario:** In this example, the measurement uncertainty is calculated using control chart data obtained from a measurement quality assurance process that mimics casework samples as closely as possible. All other conditions and assumptions are the same as in Section B, Example 2, including the use of a static weighing process.

The control chart should capture uncertainty deemed appropriate to the specific laboratory and procedure and could include factors such as environmental conditions, analysts, and sample types. A conservative approach is to select the largest standard deviation if a range of masses is charted.

**C.1 Factors contributing to weight measurement uncertainty**

As the control chart is well established, it is expected to capture all of the factors described in Example 1 except linearity and balance calibration uncertainty.

**C.2 Uncertainty Budget Table**

<table>
<thead>
<tr>
<th>Factors</th>
<th>Value (x)</th>
<th>Standard uncertainty (u), g</th>
<th>Distribution</th>
<th>Index (Relative contribution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control chart standard deviation (s)&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.0313 g</td>
<td>0.0313</td>
<td>Normal</td>
<td>84.7</td>
</tr>
<tr>
<td>Linearity&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.02 g</td>
<td>0.0116</td>
<td>Rectangular</td>
<td>11.6</td>
</tr>
<tr>
<td>Uncertainty from balance calibration report (U, coverage factor k=2)</td>
<td>0.0131 g</td>
<td>0.00655</td>
<td>Normal</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Subtotal ($\sum u_n$): 0.04945

Subtotal ($\sum (u_n)^2$): 0.002445

<sup>a</sup> $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$

<sup>b</sup> This value is the maximum permitted deviation across the mass range of the balance.

**C.3 Calculation of combined standard uncertainty**

The combined standard uncertainty per weighing event can be expressed in this example mathematically as:

$$u_c(\text{single weighing event}) = \sqrt{u(\text{control chart})^2 + u(\text{linearity})^2 + u(\text{bal cal})^2}$$

$$u_c(\text{single weighing event}) = \sqrt{(0.0313 g)^2 + (0.0116 g)^2 + (0.00655 g)^2} = 0.0340$$
As per Example 2, the total uncertainty is:

\[ u_{\text{total}} = \text{number_of_items} \times 2 \times u_c(\text{single weighing event}) \]

\[ u_{\text{total}} = 1 \text{ item} \times 2 \times 0.0340 \text{ g} = 0.0680 \text{ g} \]

**C.4 Calculation of expanded uncertainty**

The expanded uncertainty is expressed mathematically as:

\[ U = k \times u_{\text{total}} \]

Using a coverage factor \( k = 2 \) (confidence level of approximately 95% assuming the net mass follows a normal distribution):

\[ U = 2 \times 0.0680 \text{ g} = 0.136 \text{ g} \]

Using a coverage factor \( k = 3 \) (confidence level of approximately 99% assuming the net mass follows a normal distribution):

\[ U = 3 \times 0.0680 \text{ g} = 0.204 \text{ g} \]

**C.5 Results**

**C.5.1 Net Weight: 30.03 g ± 0.14 g \((k = 2)\)**

**C.5.2 Net Weight: 30.03 g ± 0.20 g \((k = 3)\)**

**D Example 4: Weighing of Multiple Items to Obtain a Total Net Weight**

**Scenario:** In this example, the laboratory must determine the net weight of a white powder, received in 15 similar plastic bags (15 items), which appear to weigh approximately 30 g each. All other conditions are the same as Example 3.

The net weight of each bag was determined as in Examples 2 and 3 (a static weighing procedure). The total net weight obtained for the powder, determined by individually placing the material from each plastic bag inside 15 separate tared weighing vessels, is 458.37 grams.

**D.1 Factors contributing to weight measurement uncertainty**

Same as Example 3. The laboratory could also choose to use the uncertainty budget approach as presented in Example 2.

**D.2 Uncertainty budget table:**

Same as Example 3. The laboratory could also choose to use the uncertainty budget approach as presented in Example 2.
D.3 Calculation of combined standard uncertainty

Same as Example 3.

\[ u_{\text{total}} = \text{number_of_items} \times 2 \times u_c(\text{single weighing event}) \]

\[ u_{\text{total}} = 15 \text{ item} \times 2 \times 0.0340 \text{ g} = 1.0200 \text{ g} \]

D.4 Calculation of expanded uncertainty

The expanded uncertainty per weighing event \((U)\) is expressed mathematically as:

\[ U = k \times u_{\text{total}} \]

Using a coverage factor \(k = 2\) (confidence level of approximately 95%):

\[ U = 2 \times 1.020 \text{ g} = 2.040 \text{ g} \]

Using a coverage factor \(k = 3\) (confidence level of approximately 99%):

\[ U = 3 \times 1.0300 \text{ g} = 3.090 \text{ g} \]

D.5 Results

D.5.1 Net Weight: 458.37 g ± 2.04 g \((k = 2)\)

D.5.2 Net Weight: 458.37 g ± 3.06 g \((k = 3)\)

E References

E.1 Books


E.2 Peer-reviewed journal articles and reviews


**E.3 On-line resources**


End of Document